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Theoretical investigations on simultaneous operation of vapour compression refrigeration cycle and Stirling cycle in miniature Stirling cooler with two-component two-phase mixture

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Abstract

The cyclic analysis of Stirling cycle working with a single gaseous fluid is modified to suit two-component two-phase mixtures. When the results were examined carefully, these showed that a substantial increase in cooling effect is obtained. In order to understand how the two-component two-phase working fluid functions in the Stirling cooler, some other processes, generally not coming into picture with a single gaseous fluid, are also considered to get good idea about the working of the cooler and then the estimates about the performance of the cooler are obtained. The processes include, simultaneous compression of gases with different ratios of C_p and C_v, the drop-wise condensation, isentropic expansion of liquid in presence of other gas, the entrainment process as observed in heat pipes, and saturation process as observed in air humidification process. The change in regenerator effectiveness due to condensation in the regenerator also has to be considered. How the above mentioned processes are affecting the system performance is discussed in detail. The analysis shows that the Stirling cycle and vapour compression cycle with isentropic expansion operate simultaneously. The criterion for selection of the gaseous carrier fluid and the condensable fluid, which undergoes phase change is established. For selection of carrier gas, helium and hydrogen are considered and for the condensable fluid, nitrogen, carbon mono-oxide, nitrogen-tri-fluoride and neon are compared. Using the above mentioned criterion, helium and nitrogen combination has been chosen for cold tip temperatures in liquid nitrogen temperature range. The paper discusses the analytical approach and the results indicate that the presence of vapour compression cycle with isentropic expansion, operating under a small difference between the condensation and evaporation temperatures, leads to high values of cooling effect and coefficient of performance (COP). It also shows that beyond a certain concentration, the cooling effect starts dropping and becomes zero at certain higher concentration. It can be mentioned that no changes in the hardware are necessary as the cooler can operate at pressures lower than normal working pressure and still provide large capacity with marginally higher power input. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The theoretical analysis of the Stirling cycle cryocoolers has been continuing over decades in order to estimate precisely the performance of the existing cryocoolers and to find ways and means to improve the performance of these units. The Stirling cycle consists of two isothermal, and two constant volume processes. The classical, first order analysis of the Stirling machines is due to Schmidt [1]. The main assumptions made in the analysis are the sinusoidal volume variations in the working spaces, no temperature gradients in the heat exchangers and same instantaneous pressure throughout the system. Walker [2] applied this first order analysis to provide what are best known as Walker's design charts. Martini [3] reported a second order method and applied it to the Stirling engine. In the second order analysis of the Stirling coolers, ideal refrigeration effect and ideal power requirement are first computed based on Schmidt's analysis. Each identifiable loss is calculated separately and it is assumed that sum of these losses can be used to obtain the net cooling capacity and also the power requirement. Walker et al. [4] applied Martini's second order analysis to the Philips machine, PPG-102 and the results were in agreement with the experimental results within reasonable tolerance limit. They assumed

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the processes of compression and expansion to be adiabatic. However, in practice the compression process tends to be adiabatic and the expansion occurs under isothermal condition. Atrey et al. [5] simulated the Stirling cryogenerators considering isothermal expansion and adiabatic compression processes. The analysis when applied to PPG-102 showed better matching with experimental results of Walker [4], thus indicating validity of assuming isothermal expansion and adiabatic compression. Generally, all these analysis were carried out for a single phase (gaseous) working fluid. Based on the thermodynamic and transport properties hydrogen or helium have been preferred as working fluid.

The use of two-component two-phase working fluid in the Stirling cryocooler was first discussed by Walker [6]. The main conclusion was that the performance of the system would improve if a two-component two-phase working fluid is used. Metwally and Walker [7] followed this work with chemically reactive working fluid for the Stirling engine. They indicated that a substantial increase in net cycle work can be obtained without any penalty on size, weight and cost of the engine.

Patwardhan and Bapat [8] have discussed the cyclic simulation of the Stirling cryogenerator with a particular two-component two-phase working fluid combination. They gave a comparison of helium and hydrogen as the gaseous fluid used with nitrogen as the condensing fluid [9]. The analysis showed that the use of two-component two-phase working fluid does not necessarily improve the performance of the Stirling cryocooler compared to the performance with single phase gaseous fluid and the thermodynamic and thermophysical properties of the mixture become important to decide whether the mixture will be better or not.

The present work modifies the analysis used for the cryogenerators for the application to a miniature, free piston, free displacer, linear motor driven, opposed piston compressor, split Stirling cryocooler. The displacer is attached to a mechanical spring and is non-motorised. Hence the stroke and phase shift are dependent on the spring design and operating pressure. Under the resonance conditions, the movement of the displacer will be maximum and equal to the design value of the displacer stroke. Hence it becomes necessary to modify the above analysis used for the cryogenerators.

2. Approach of the present work

The working fluid is considered to be a mixture of two chemically non-reacting fluids. One of the fluids used is assumed to behave as a perfect gas in the complete range of temperatures and pressures and is referred to as the carrier gas. The other component goes through a phase change process depending on the temperature and pressure conditions but is in gaseous phase at ambient temperature and pressure conditions and is referred as the condensable fluid. In order to understand how two-component two-phase working fluid functions in the Stirling cooler, some other processes, generally not occurring with a single gaseous fluid, are considered briefly to have precise idea about the working mechanism inside the cooler.

2.1. Simultaneous compression of mixture of gases

In isentropic compression process, the temperature at the end of compression process depends on the ratio of specific heats at constant pressure and constant volume. If the working fluids have different specific heat ratios and the ratio for the condensable fluid is less than that for the carrier gas, the temperature at the end of compression of mixture, for a given pressure ratio, will be less than the temperature obtained when only carrier gas is used as the working fluid. This will contribute to lowering the maximum temperature in the cycle. However, if the ratio for condensable fluid is same or higher than that for the carrier gas, such benefit will not be available.

2.2. Condensation process and heat transfer

In a continuous condensation process e.g., over the condenser of vapour compression refrigeration system, the process starts with drop-wise condensation, then a film gets formed by merging of large droplets and then the process continues as a film condensation process. However, if the condensation occurs in a cyclic fashion i.e., the condensate gets formed over a surface and removed from the surface almost immediately after formation, the surface will remain clean. Further, with a very small quantity of condensate distributed over a very large surface area the droplet size will be so small that the film may not get formed. As mentioned by Hausen [10], it is very difficult to get drop-wise condensation on a given surface continuously and so the film condensation is generally considered for the purpose of heat transfer coefficient during phase change process from vapour phase to liquid phase. He also mentions that if it is possible to create the conditions by which one can get drop-wise condensation, the heat transfer coefficients can be of the order of 80 000 W/(m^2 K) or even higher. With such large heat transfer coefficient, the surface area required will be greatly reduced compared to that for film condensation. When only carrier gas is considered as the working fluid, the resulting heat transfer coefficients, from the correlations generally used, are between 30 000–40 000 W/(m^2 K). This information indicates that the use of a condensable fluid in the Stirling cooler will improve the regenerator effectiveness.

2.3. Entrainment process

Towards the cold end of the regenerator, the condensation will occur and when the displacer moves away from the cold tip, it will draw the vapour from the cold end of the regenerator. At the interface between the regenerator matrix surface and the vapour, the vapour will exert a shear force on the liquid droplets on the regenerator matrix surface. The magnitude of the shear force will depend on the vapour properties and its velocity and its action will be to entrain droplets of liquid and transport them to the expansion space. This action will be resisted by the surface tension in the liquid. Thus, the Weber number (We) defined as the ratio of the inertial force of vapour and surface tension force of liquid, provides a convenient measure of the likelihood of entrainment. The expression for Weber number used here is

$$\mathrm{We} = (\text{vapour density}) \times (\text{vapour velocity}^2) \times (\text{characteristic dimension})/(\text{surface tension}).$$

If Weber number equals or exceeds 1.0, it is assumed that entrainment will occur. In case of the present day miniature cryocoolers, the total stroke of the displacer is of the order of 4–8 mm or so and the frequency at which these are operated is around 50 Hz. The resulting high, average linear speed will almost always ensure the entrainment of liquid by vapour.

2.4. Vapour compression cycle with condensable fluid

Fig. 1 shows a schematic diagram of general vapour compression cycle on pressure–enthalpy diagram. The expansion process in the practical systems, such as household refrigerators, is isenthalpic and a throttle valve or a capillary tube is employed for the purpose. The degree of superheat is 10–15°C in these systems. However, if we consider the cycle to be working within

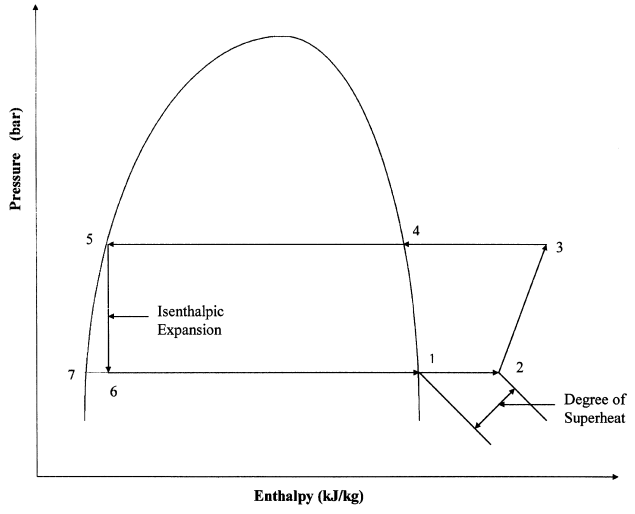

Fig. 1. Schematic diagram of general vapour compression cycle.

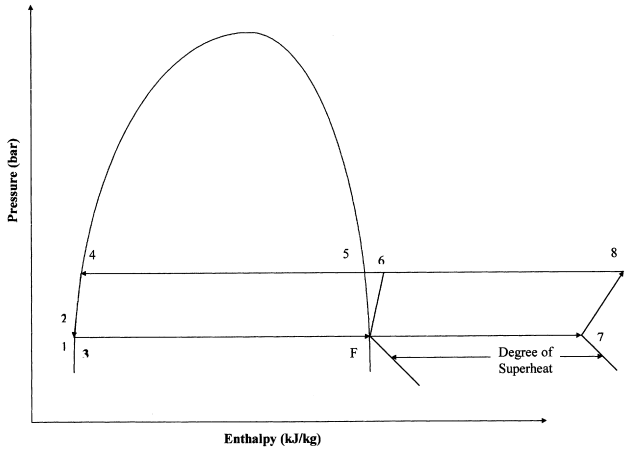

Fig. 2. Schematic diagram of vapour compression cycle with large superheating, low pressure ratio and isentropic expansion.

the cooler, with condensable fluid as the working medium, with a small difference in condensation and evaporation temperatures, with a large superheat and isentropic expansion process replacing isenthalpic expansion process, the cycle will look as shown in Fig. 2. If the latent heat of the condensable fluid is large, the cooling effect available with a small mass of condensable fluid also will turn out to be large. Further, if the vapour pressure at the required cold temperature is large, a larger concentration can be used which will further increase the cooling effect due to increase in mass of condensable fluid expanded and evaporated in the expansion space. Compared to this large cooling effect available, the work input for the isentropic compression process with a small pressure ratio will be very small and thus result in high coefficient of performance (COP). This will help in improving the COP of the cooler as a whole.

2.5. Saturation process

Fig. 3 shows the saturation process generally used in air humidification process on temperature–entropy diagram. At constant temperature, if the vapour is added to vapour existing at state 1, the partial pressure will increase. This process of addition of vapour can continue till the state 2 is reached where the space is saturated with vapour. Any addition of vapour beyond state 2 will result in condensation and the overall state will be wet vapour as shown by state 3 in the liquid–vapour region on the temperature–entropy diagram.

2.6. Variation in heat exchanger effectiveness in part of the regenerator from where condensation begins

As the condensation starts, the mass of working fluid in the vapour phase will reduce linearly for the further sections till the cold end of the regenerator is reached. If

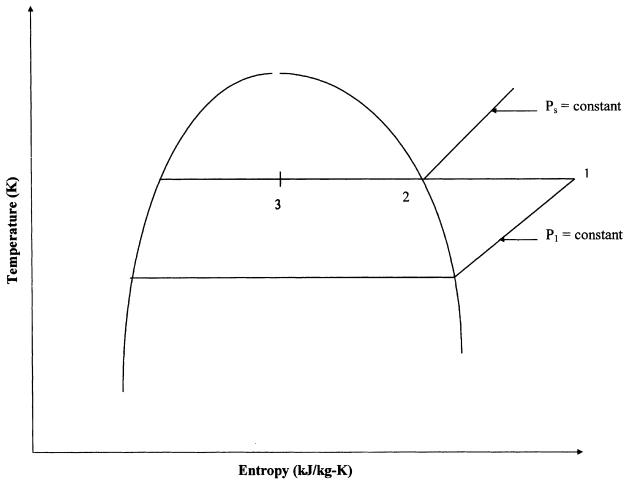

Fig. 3. Saturation process on schematic temperature–entropy diagram for the condensable fluid.

the temperature of matrix at a particular section is lower than the saturation temperature, the liquid will become subcooled liquid and will act similar to matrix material for the next lot of vapours moving over this surface. As the effectiveness of the regenerator increases with the increase in the ratio of heat capacities of the matrix and the fluid, this will tend to marginally increase the effectiveness. Second, if the drop-wise condensation occurs, as mentioned earlier, the heat transfer coefficient will be increasing by almost a factor of 2 or even more. This again will tend to increase the effectiveness of the regenerator substantially. Third, the heat of condensation forms the additional heat load on the regenerator and hence will tend to reduce the regenerator effectiveness. As the total heat transfer load on the regenerator, even in case of a single fluid is reported to be of the order of 100 times or more compared to the cooling capacity and as the heat load due to condensation will at most be only 4–5 times the cooling capacity, this additional load due to condensation should not be changing the regenerator effectiveness substantially.

Thus, with the above three effects working concurrently, it can safely be assumed that the change in effectiveness will be negligible compared to that with the single fluid under the same conditions of mean pressure.

2.7. Mixing rules for thermodynamic and thermophysical properties

The analysis needs the thermodynamic and thermophysical properties of mixture in the vapour phase at different steps. The properties needed are, e.g., dynamic viscosity, thermal conductivity, heat capacity, etc. These are calculated using simple mixing rule wherein the contribution for a required property by a particular component is proportional to its composition in the vapour phase of the mixture.

3. Analysis of the two-component two-phase system

The complete cycle is split into suitable number of intervals. The start of the cycle can be considered from any point but it is explained here starting with the compression process. The hot compressed gaseous mixture enters the regenerator and gives heat to the regenerator matrix material. At the low temperature end of the regenerator the condensable fluid changes its phase from vapour to liquid. During the expansion stroke, as the displacer moves away from the cold tip, the suction occurs and simultaneously the pressure level continues to drop due to increasing volume of the expansion space. The condensed fluid is entrained by the carrier gas in to the expansion space. Depending on the composition a certain quantity of the condensable fluid goes through the reverse phase change in the presence of the carrier gas till the saturation condition occurs. The carrier gas continues to act as in single fluid system and also provides some cooling effect due to its pressure–volume variation. As an overall effect, for a correct composition the cooling capacity should increase.

3.1. Assumptions in the present analysis

The following are considered as the important assumptions in the present analysis:

1. The movements of the piston and the displacer are sinusoidal.
2. The expansion and compression processes are isothermal and adiabatic, respectively.
3. The carrier gas behaves as a perfect gas.
4. The vapour of the condensable fluid also behaves as a perfect gas.
5. The total pressure is the sum of the partial pressures of carrier gas and the condensable fluid and is essentially the same in all the system components at any instance.
6. The fraction of condensable fluid in liquid phase does not contribute to total pressure.
7. The volume of the condensable fluid fraction in liquid phase is negligible compared to regenerator matrix void volume.
8. At the end of expansion stroke, the working fluid in the expansion space is in vapour phase.
9. The quasi-steady state is attained and hence the volumes and pressures are subject only to cyclic variations.

4. Computer program

Fig. 4 shows the flow chart of the computer program developed. Complete information on materials of construction, the dimensional data of all the working spaces, the ambient temperature, the mean operating

pressure, the frequency, the composition of the gas mixture, the desired temperature at the cold tip of the cryocylinder, the regenerator matrix details, the strokes of piston and the displacer and the phase difference between the two are required, to estimate the cooling capacity available from a given cryocooler. The cycle is split in to the desired number of angular intervals.

Thus, the analysis starts with the calculation of volume variations of working spaces with crank angle. Using the thermodynamic relations, and the Gauss–Siedel iterative technique discussed by Atrey et al. [5], the pressure and temperature variation in the compression space is calculated. It is initially assumed that one mole of the gas mixture is charged in the system. For the two component mixture, the total pressure is the sum of the partial pressures of the two components. The mean pressure with one mole of gas is calculated. If the number of moles of gas in the system is changed, the mean pressure also will change accordingly. Thus, the actual mass of gas mixture depends on the mean operating pressure considered in the cycle. Using the ratio of the actual mean pressure in the cycle and the mean pressure with one mole of gas mixture, the values of the pressures at different crank angles are modified to give actual values.

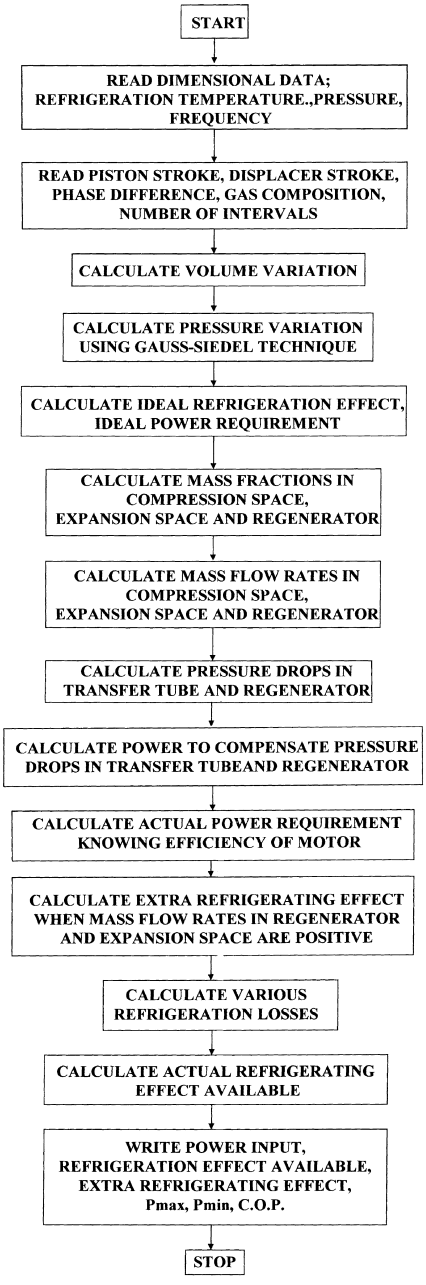

Fig. 4. Flow chart of the simulation program developed.

Having the volume, pressure and temperature variation with the crank angle known, the mass fractions in the various spaces are calculated for each interval. Using the difference in mass fractions between the interval under consideration and the next interval, the molecular weight of the working fluid and the speed of the cooler, the mass flow rates for the expansion space, the regenerator and the compression space are calculated for a given interval.

4.1. Ideal power requirement

The algebraic sum of the product of pressure and volume difference in compression space for each interval gives the ideal power input. The pressure drops in the various components viz., the tube connecting the compression space and the displacer unit and the regenerator are calculated. This is done knowing the mass flow rates for the compression space, the regenerator and the expansion space and using the standard formulae mentioned by Martini [3]. The power requirement to compensate these pressure drops is added to the ideal power requirement to give the theoretical power requirement of the cryocooler.

4.2. Ideal refrigerating effect due to pressure–volume variation

Similar to the power requirement calculation, the algebraic sum of the product of pressure and the volume difference for expansion space for each interval gives the ideal refrigerating effect. However, this holds good only when we consider that cooling effect is the result only of pressure and volume changes which is true when the working fluid is a single phase, gaseous mixture. The effect of the phase change is not reflected through this approach of calculations and needs to be considered separately.

4.3. Refrigeration effect through phase change process

In the program, it is assumed that the condensed working fluid is drawn into the expansion space when

the displacer moves away from cold end and during phase change from liquid to vapour phase the latent heat required is supplied through the heat load on the cold tip of the cryocylinder.

4.4. Overall refrigeration effect

Thus, the total cooling effect available is due to pressure volume variation of gas mixture in vapour phase and additional cooling effect available due to phase change. The loss in refrigeration effect is due to regenerator ineffectiveness, shuttle heat conduction, temperature swing, pumping action, instantaneous pressure drop, and heat conduction through solid components involved in the system. As mentioned earlier, in the second order analysis, these losses are calculated and added together to provide the total loss. The difference between the total available refrigerating effect due to pressure–volume variation and phase change process and, the above losses provides the theoretically calculated overall refrigeration capacity of a given cooler. Knowing the refrigeration capacity and the power requirement, the COP and refrigerating efficiency can be obtained.

5. Criterion for selecting working fluids

5.1. Carrier gas

There is no specific information available in the open literature about the selection of the working fluids. However, in the single gaseous fluid systems, for the liquid nitrogen temperature range, only one main criteria seems to be considered by most researchers viz., the fluid should remain in the gaseous phase under all conditions that may occur in the system. As such, only hydrogen and helium are generally used in the Stirling systems.

5.2. Condensable fluid

An attempt has been made to decide on the criterion for selecting the working fluid based on the following properties.

Latent heat during phase change: The latent heat during phase change should be as high as possible. This will reduce the mass of condensable fluid to provide the desired additional cooling effect and still allow a large quantity of the carrier gas in the system.

High liquid density: High liquid density will ensure that the volume of condensable fluid after the phase change will be small (negligible) compared to the void volume of the regenerator which will ensure that no blockage of the regenerator will occur at any stage in the operation of the system.

Specific heat in gaseous phase: It should have a low specific heat so that the heat load on the regenerator will be small. It will be desirable to have this value lower than the carrier gas to be used in the system.

Thermal conductivity: In order to support good heat transfer in the regenerator as well as in the expansion space, the higher thermal conductivity of the condensable fluid is beneficial.

Dynamic viscosity: In order to have smooth flow and reduced pressure drops in the system components, it is desirable to have low dynamic viscosity of the condensable fluid.

Other criterion: The other general criterion for rejection rather than selection will be the normal boiling point, the critical point temperature and pressure, chemical reactivity with the system component materials, handling problems and cost of the condensable fluid.

6. Results and discussion

Patwardhan and Bapat [9] have compared helium–nitrogen and hydrogen–nitrogen as two-component two-phase combinations for use in Philips Stirling machine PLN-106. It was concluded that while helium–nitrogen combination improves the performance of the machine, the performance deteriorates with the use of hydrogen–nitrogen mixture when compared with the performance of single working fluids helium and hydrogen, respectively. The thermophysical properties of the mixtures were cited as the reason for this type of behaviour.

Based on the above criterion, helium is used as the carrier gas. Using the criterion discussed above for selection of condensable fluid, neon, nitrogen-tri-fluoride, carbon mono-oxide and nitrogen were considered and nitrogen was selected as the condensable fluid for the liquid nitrogen temperature range. A typical cooler, like e.g., the SL-100 made by AIM, Germany is chosen for the theoretical analysis. This specific unit considered could provide 1 W cooling capacity at the temperature of 74.7 K when charged with helium at 20 bar.

6.1. Results of the present work

The computer program was run with the dimensional and other data as mentioned above for various concentrations of nitrogen–helium gas mixtures. The program was first run for helium alone as the working fluid and it was observed that the cooler operation was matching with the resonance conditions for piston and displacer motions. As the cooler will work, in general, under conditions other than the resonance conditions and as it is free piston, free displacer cooler with non-motorised displacer, it is not possible to predict any of the three parameters, viz., piston stroke, displacer stroke

and the phase difference between them. Hence, the same conditions, which give the proper matching with helium alone, are chosen also as the parameters for the gas mixture, to see the effect of composition of the working fluid. As for helium, so also for the mixtures, the total loss is subtracted only from the refrigerating effect available due to pressure–volume variation in the expansion space.

Fig. 5 shows the variation of cooling capacity available as a function of gas composition. It is observed that the cooling capacity available only due to pressure–volume variation decreases continuously. The decrease in cooling capacity can be explained as follows.

As the condensable fluid is added to the working fluid, some fraction of the condensable fluid condenses and thus reduces the mass of the working fluid in gaseous phase which is responsible to provide the cooling effect due to pressure–volume variation. The condensate evaporates during the expansion stroke and contributes to total pressure in the expansion space. As the vapour formation takes place right up to the end of the displacer stroke, the pressure drop for the carrier gas in presence of the vapours will be less than what would have occurred if it were a single gaseous fluid. Thus, the cooling effect due to pressure–volume variation decreases.

Fig. 5 also shows the variation of additional cooling effect available solely due to phase change as a function of working fluid composition. Initially the increment in concentration was taken as 0.5%. It was found that the additional cooling effect suddenly dropped from a high value at 2.0% to zero at 2.5%. This sudden change did not seem possible and hence the program was run at much smaller concentration increments from concentration of 2.0% onwards. It was found that the cooling

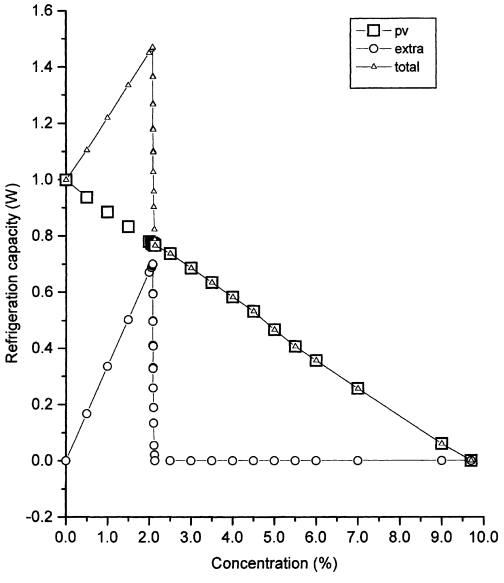

Fig. 5. Variation of refrigeration capacity with concentration of condensable fluid.

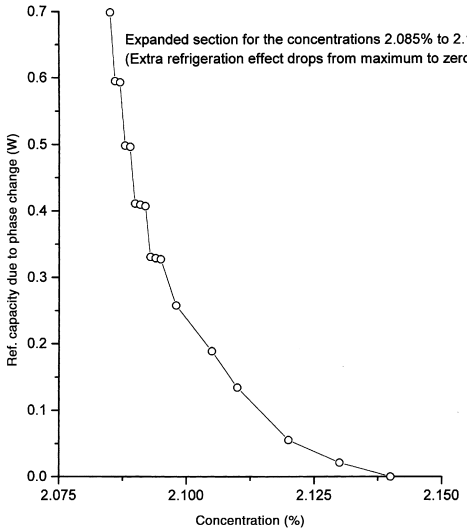

Fig. 6. Expanded section for the concentrations 2.085–2.14%.

capacity continues to increase till the concentration reaches the value of 2.085%. This is the maximum additional cooling effect available. It was also observed that the partial pressure exactly reaches the saturation pressure of the condensable fluid at the expansion space temperature. For concentrations higher than 2.085% this value of partial pressure is reached before the end of expansion stroke. As the expansion space gets saturated with the condensable fluid, no further evaporation of condensable fluid is possible. A stage comes when the above mentioned partial pressure exists at the beginning of the expansion stroke itself and as no evaporation is possible, the additional cooling effect reduces to zero and some quantity of condensed fluid remains stored in liquid phase in the regenerator. This part of Fig. 5 is shown (for concentration range 2.085–2.140%) in an enlarged form in Fig. 6. Beyond this composition a larger and larger quantity remains in liquid phase throughout the cycle. In other words, the actual operating fluid quantity in vapour phase decreases, the pressure in the system starts dropping and leads to reduction in the cooling capacity due to pressure–volume variation also.

7. Conclusions

The present work explains the various processes actually taking place in the Stirling cooler with two-component two-phase mixture. It has been made clear that the improvement in the performance is due to in-built vapour compression cycle working on condensable fluid alone. The criterion for selection of condensable fluid for a particular temperature range is established. Finally, it can be said that substantial improvement in

the capacity, without making any modifications in the system hardware is also possible.

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